12

EDGEWOOD
RESEARCH, DEVELOPMENT & ENGINEERING CENTER
U.S. ARMY CHEMICAL AND BIOLOGICAL DEFENSE COMMAND

ERDEC-TR-207

# ELECTROMAGNETIC SCATTERING BY MAGNETIC SPHERES: THEORY AND ALGORITHMS



Merrill E. Milham

RESEARCH AND TECHNOLOGY DIRECTORATE

October 1994





CUM SCIENTIA DEFENDIMUS

Aberdeen Proving Ground, MD 21010-5423

# Disclaimer

# REPORT DOCUMENTATION PAGE

*Form Approved*
*OMB No. 0704-0188*



**1. AGENCY USE ONLY** *(Leave blank)*

**2. REPORT DATE**
1994 October

**3. REPORT TYPE AND DATES COVERED**
Final, 93 Jul - 94 Jan

**4. TITLE AND SUBTITLE**
Electromagnetic Scattering by Magnetic Spheres: Theory and Algorithms

**5. FUNDING NUMBERS**
PR-1O162622A552

**6. AUTHOR(S)**
Milham, Merrill E.

**7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)**
DIR, ERDEC, ATTN: SCBRD-RTE, APG, MD 21010-5423

**8. PERFORMING ORGANIZATION REPORT NUMBER**
ERDEC-TR-207

**9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)**

**10. SPONSORING/MONITORING AGENCY REPORT NUMBER**

**11. SUPPLEMENTARY NOTES**

**12a. DISTRIBUTION/AVAILABILITY STATEMENT**
Approved for public release; distribution is unlimited.

**12b. DISTRIBUTION CODE**

**13. ABSTRACT** *(Maximum 200 words)*

The theory for the scattering of magnetic spheres is developed by means of scaling functions. This theory leads in a natural way to the development of scattering algorithms which use exponential scaling to overcome computational overflow problems. The design and testing of the algorithm is described. Fortran codes which implement the algorithmic design are presented and examples of code use are given. Listings of the code are included.



**14. SUBJECT TERMS**

| | | |
|---|---|---|
| Mueller matrix | Scaling functions | Electromagnetic scattering |
| Magnetic sphere | Complex permittivity | |
| Bessel functions | Complex permeability | |

**15. NUMBER OF PAGES**
56

**16. PRICE CODE**

| 17. SECURITY CLASSIFICATION OF REPORT | 18. SECURITY CLASSIFICATION OF THIS PAGE | 19. SECURITY CLASSIFICATION OF ABSTRACT | 20. LIMITATION OF ABSTRACT |
|---|---|---|---|
| UNCLASSIFIED | UNCLASSIFIED | UNCLASSIFIED | UL |

Blank

# PREFACE

The work described in this report was authorized under Project No. 1O162622A552, Smoke/Obscurants. This work was started in July 1993 and completed in January 1994.

The use of trade names or manufacturers' names in this report does not constitute an official endorsement of any commercial products. This report may not be cited for purposes of advertisement.

This report has been approved for release to the public. Registered users should request additional copies from the Defense Technical Information Center; unregistered users should direct such requests to the National Technical Information Service.

| Accesion For | |
|---|---|
| NTIS CRA&I | ☒ |
| DTIC TAB | ☐ |
| Unannounced | ☐ |
| Justification | |
| By | |
| Distribution / | |
| Availability Codes | |
| Dist | Avail and / or Special |
| A-1 | |

Blank

CONTENTS

Blank

# ELECTROMAGNETIC SCATTERING BY MAGNETIC SPHERES: THEORY AND ALGORITHMS

## 1. INTRODUCTION

The scattering of electromagnetic radiation by spheres is a classic physics problem for which the theoretical solution has been known for well over fifty years. The development of the theory for the scattering of light by a sphere, which culminated in the general solution given by Gustav Mie,[1] dates to the late nineteenth and early twentieth centuries. Usually the Mie theory is formulated to describe the scattering for materials for which the magnetic permeability ($\tilde{\mu}$) is one. Kerker[2] has explored a number of the unusual scattering effects for magnetic spheres. The approach taken here is to develop the theory and algorithms for scatterers with both a complex permittivity ($\tilde{\epsilon}$) and a complex permeability; the usual Mie solution for a homogenous, nonmagnetic spherical scatterer will then appear as a special case of this more general theory.

The computational history of these scattering problems is much shorter than that of the formal solution since scattering computations flourished only with the development of electronic computers in latter half of this century. The number of scattering calculations carried out before the modern computer era was understandably small since the computational labor involved in evaluating scattering functions such as Ricatti-Bessel functions for complex arguments was extreme. Electronic computation brought problems of its own: early computer algorithms were plagued with numeric difficulties that stemmed principally from ill-conditioning produced by subtraction of nearly equal numbers and instability in recursion relations. Gradually several generally reliable scattering subroutines appeared;[4-6] these Mie scattering codes have been the mainstay of researchers in the atmospheric sciences over the past two decades.

Advances in computational technique have led to the development of a general purpose subroutine package[7] for computing Bessel functions of complex argument and nonnegative order. These subroutines fill a void in complex Bessel function software and implement features such as multiple computational schemes, exponential scaling, and internal error checking including underflow and overflow detection, which until now have not been incorporated in electromagnetic scattering codes. The purpose of this paper is to formulate the electromagnetic scattering problem and computational algorithms for magnetic spheres to take maximum advantage of the features of these general purpose Bessel function routines.

In what follows the theory for the scattering of electromagnetic radiation by magnetic spheres is developed by means of scaling functions. This theory leads in a natural way to the development of scattering algorithms which use exponential scaling to overcome computational overflow problems. The design and testing of the algorithm is described. Finally, a Fortran code which implements the algorithmic design is presented and examples of its usage are given.

## 2. ELECTROMAGNETIC SCATTERING THEORY

For spheres, the efficiencies (ratio of the optical cross section to the geometric cross section) for extinction, $Q_e$, scattering, $Q_s$, absorption $Q_a$, backscatter, $Q_b$, and the amplitude functions ($S_1$, $S_2$) are:[8]

$$Q_e = \frac{2}{X^2} \sum_{n=1}^{\infty} (2n+1)\,\mathrm{Re}\,(a_n + b_n) \tag{1}$$

$$Q_s = \frac{2}{X^2} \sum_{n=1}^{\infty} (2n+1)[\,|a_n|^2 + |b_n|^2\,] \tag{2}$$

$$Q_a = Q_e - Q_s \tag{3}$$

$$Q_b = \frac{1}{X^2} \left| \sum_{n=1}^{\infty} (-1)^n (2n+1)(a_n - b_n) \right| \tag{4}$$

$$S_1(\theta) = \sum_{n=1}^{\infty} \frac{(2n+1)}{n\,(n+1)} [a_n \pi_n(\mu_1) + b_n \tau_n(\mu_1)] \tag{5}$$

$$S_2(\theta) = \sum_{n=1}^{\infty} \frac{(2n+1)}{n\,(n+1)} [a_n \tau_n(\mu_1) + b_n \pi_n(\mu_1)] \tag{6}$$

The angular scattering functions ($\pi_n$, $\tau_n$) corresponding to the scattering angle ($\theta$) are defined in terms of the Legendre polynomials ($P_n$)[9] as:

$$\pi_n(\mu_1) = P'(\mu_1) \tag{7}$$

$$\tau_n(\mu_1) = \mu_1 \pi_n(\mu_1) - (1-\mu_1^2)\pi_n'(\mu_1) \tag{8}$$

where $\mu_1 = \cos(\theta)$. All of which are given in terms of the scattering coefficients ($a_n$, $b_n$):[2]

$$a_n = \frac{\tilde{\mu}\, U_n^s(m,X) - m\, V_n^s(m,X)}{\tilde{\mu}\, W_n^s(m,X) - m\, Z_n^s(m,X)} \tag{9}$$

$$b_n = \frac{m\, U_n^s(m,X) - \tilde{\mu}\, V_n^s(m,X)}{m\, W_n^s(m,X) - \tilde{\mu}\, Z_n^s(m,X)} \tag{10}$$

where

$$U_n^s(m,X) \equiv \psi_n(X)\,\psi_n'(mX)$$
$$V_n^s(m,X) \equiv \psi_n(mX)\,\psi_n'(X)$$
$$W_n^s(m,X) \equiv \zeta_n(X)\,\psi_n'(mX) \qquad (11)$$
$$Z_n^s(m,X) \equiv \psi_n(mX)\,\zeta_n'(X)$$

$\psi_n$, $\zeta_n$ are the Ricatti-Bessel functions and are defined in terms of the nth order spherical Bessel functions of the First and Second Kind ($j_n$, $y_n$) as $\psi_n(z) = zj_n(z)$ and $\zeta_n(z) = z[j_n(z) - iy_n(z)]$.[9] $X = \pi D/\lambda$ is the size parameter where, $D$, is the diameter; $\lambda$, the wavelength; and $m = \sqrt{\tilde{\mu}\tilde{\epsilon}}$, the refractive index for which $\tilde{\mu}$, the permeability, and $\tilde{\epsilon}$, the permittivity, are complex quantities. Primes denote derivatives with respect to the argument.

It will now be shown that the scattering coefficients can be expressed directly in terms of the Bessel functions of half-integer order only. The Riccati-Bessel functions are related to the spherical Bessel functions as described above, and the spherical Bessel functions are related to the half-integer order Bessel functions by:[9]

$$j_n(z) = \sqrt{\frac{\pi}{2z}}\,J_{n+1/2}(z)$$
$$y_n(z) = \sqrt{\frac{\pi}{2z}}\,Y_{n+1/2}(z) \qquad (12)$$

The above equations can be used with recurrence relations for derivatives[9] and (11) to find the scaled functions:

$$\tilde{U}_n^s(m,X) = J_{n+1/2}(X)\,J'_{n+1/2}(mX)$$
$$\tilde{V}_n^s(m,X) = J_{n+1/2}(mX)\,J'_{n+1/2}(X)$$
$$\tilde{W}_n^s(m,X) = H^{(2)}_{n+1/2}(X)\,J'_{n+1/2}(mX) \qquad (13)$$
$$\tilde{Z}_n^s(m,X) = J_{n+1/2}(mX)\,H^{(2)\prime}_{n+1/2}(X)\,.$$

The set of all scaled functions ($\tilde{g}_1$) is related to the set of all unscaled functions ($g_s$) by:

$$\tilde{g}_1 = \left\{ \tilde{U}_n^s(m,X),\ \tilde{V}_n^s(m,X),\ \tilde{W}_n^s(m,X),\ \tilde{Z}_n^s(m,X) \right\}$$

$$= \left\{ \frac{2}{\pi X \sqrt{m}} g_s \right\} \tag{14}$$

where

$$g_s \equiv \left\{ U_n^s(m,X),\ V_n^s(m,X),\ W_n^s(m,X),\ Z_n^s(m,X) \right\}.$$

Since the sphere scattering coefficients are invariant when the scaled functions, $\tilde{g}_1$, are used to compute them, the scattering coefficients may be expressed in the form:

$$a_n = \frac{\sqrt{\tilde{\mu}}\, J_{n+1/2}(X) J'_{n+1/2}(mX) - \sqrt{\tilde{\epsilon}}\, J_{n+1/2}(mX) J'_{n+1/2}(X)}{\sqrt{\tilde{\mu}}\, H^{(2)}_{n+1/2}(X) J'_{n+1/2}(mX) - \sqrt{\tilde{\epsilon}}\, J_{n+1/2}(mX) H^{(2)\prime}_{n+1/2}(X)} \tag{15}$$

$$b_n = \frac{\sqrt{\tilde{\epsilon}}\, J_{n+1/2}(X) J'_{n+1/2}(mX) - \sqrt{\tilde{\mu}}\, J_{n+1/2}(mX) J'_{n+1/2}(X)}{\sqrt{\tilde{\epsilon}}\, H^{(2)}_{n+1/2}(X) J'_{n+1/2}(mX) - \sqrt{\tilde{\mu}}\, J_{n+1/2}(mX) H^{(2)\prime}_{n+1/2}(X)} \tag{16}$$

This scaling of the scattering coefficients is referred to as scaling of the first kind; in developing the algorithms for computing the scattering by spheres a scaling of the second kind will be employed.

## 3. SCATTERING ALGORITHMS

Based on Equations 15 and 16 and a set of subroutines[7] that compute Bessel functions for complex arguments and nonnegative order, the development of algorithms for computing the scattering by spheres will be described.

One of the first decisions to be made when constructing a scattering algorithm is the number of terms to be included in the sums for cross sections and scattered amplitudes. Dave[4] adopted a dynamic procedure for terminating the Mie series summations, which as pointed out by Wiscombe[10] can fail in some circumstances. Wiscombe[4,10] replaced Dave's

procedure for estimating the number of terms in the Mie series summations with an improved *a priori* calculation of the number of terms required for convergence ($N$):

$$N = X + c\,X^{\frac{1}{3}} + n_m\,, \tag{17}$$

where $c$ and $n_m$ are numeric constants, and $N$ is the largest integer that does not exceed the value of $N(X)$. Bohren and Huffman[5] employed the same type of function for sphere calculations. This type of estimate was first suggested, without the "$n_m$" term, by Khare.[11] The practical significance of $n_m$ is to provide a lower bound on the number of summation terms. $n_m$ becomes the total $N$ when the integer part of $X + cX^{1/3} < 1$.

Based on numerical tests of sphere scattering codes for size parameters in the range $0.001 \leq X \leq 5000$ the effective minimum number of Bessel function orders is set at three, and $c = 4$. This corresponds to $X_{min} \approx 1.5 \times 10^{-2}$. In practice this was accomplished by setting the number of Bessel function orders (starting with order zero) by using a function as specified by Equation 15 for $c = n_m = 4$. The zero order functions are not used directly in the Mie sums, but are used to find the Bessel function derivatives.

As pointed out by Dave,[3] computation of the scattering coefficients for complex values of the refractive index leads to potential overflow conditions since functions that increase exponentially with $kX$ are required. The standard method for overcoming this computational problem is to employ logarithmic derivatives of the Ricatti-Bessel functions ($\psi'_n/\psi_n$, $\zeta'_n/\zeta_n$), a technique that dates back to Infield[12] (1947). Here another approach has been taken to this problem: the use of exponential scaling.

Scaled values of the Bessel functions appearing in the unified expressions for the scattering coefficients (Equations 13 and 14) are available as an option in the Amos[7] subroutine package; these functions are defined as:

$$\begin{aligned} J^{\sigma}_{n+\frac{1}{2}} &= e^{-|kX|}\,J_{n+\frac{1}{2}} \\ Y^{\sigma}_{n+\frac{1}{2}} &= e^{-|kX|}\,Y_{n+\frac{1}{2}}\,. \end{aligned} \tag{18}$$

This gives rise to a scaling of the second kind for which:

$$\begin{aligned} \bar{g}_2 &= \left\{ \tilde{U}_n(m,X),\ \tilde{V}_n(m,X),\ \tilde{W}_n(m,X),\ \tilde{Z}_n(m,X) \right\} \\ &= \left\{ e^{-|kX|}\,g \right\} \end{aligned} \tag{19}$$

where

$$g \equiv \left\{ U_n(m,X),\ V_n(m,X),\ W_n(m,X),\ Z_n(m,X) \right\}.$$

$U_n$, $V_n$, $W_n$, and $Z_n$ are the previously defined functions.

The scattering coefficients are invariant to any combination of these scalings so that no algorithmic compensation for scaling is required. Exponential overflow and underflow limits for the Bessel function package are machine dependent quantities that are defined as:

$$\begin{aligned} e^{\pm\delta} &= B^{\pm K} \,/\, 10^{\pm 3} \\ e^{\pm\epsilon} &= (e^{\delta} \cdot \Delta)^{\pm 1} \end{aligned} \tag{20}$$

where $\delta$ = overflow or underflow limits; epsilon = near - overflow or near - underflow limit; B = number base; K = maximum exponent; and $\Delta$ = unit round - off. Overflow quantities are produced by the upper signs; underflow quantities are produced by the lower signs. An offset of $10^3$ is included to allow for some impreciseness in tests. The near-overflow condition is a convenient criterion for shifting from unscaled to scaled Bessel functions; for $|kX| > \epsilon$ scaled functions are used.

Computation of the angular scattering presents no particular difficulties. For the scattered amplitudes the procedure advocated by Wiscombe[10] has been adopted. In this procedure scattered amplitudes are cumulated in a loop over the summation index, n, which is nested within a loop over the scattering angles. The arithmetic is more efficient if the quantities:

$$\begin{aligned} S^{+}(\mu_1) &= S_1(\mu_1) + S_2(\mu_1) \\ S^{-}(\mu_1) &= S_1(\mu_1) - S_2(\mu_1) \end{aligned} \tag{21}$$

are computed. $S_1$ and $S_2$ are then computed outside of the n-loop from:

$$\begin{aligned} S_1(\mu_1) &= \tfrac{1}{2}\,[\,S^{+}(\mu_1) + S^{-}(\mu_1)\,] \\ S_2(\mu_1) &= \tfrac{1}{2}\,[\,S^{+}(\mu_1) - S^{-}(\mu_1)\,]\,. \end{aligned} \tag{22}$$

The angular functions are computed from the recursion relations:[10]

$$\pi_{n+1}(\mu_1) = \mu_1\, \pi_n(\mu_1) + \left(\frac{n+1}{n}\right)[\,\mu_1 \pi_n(\mu_1) - \pi_{n-1}(\mu_1)\,] \tag{23}$$

$$\tau_n(\mu_1) = n\,[\,\mu_1\, \pi_n(\mu_1) - \pi_{n-1}\,] - \pi_{n-1}(\mu_1) \tag{24}$$

The usual procedure is to initialize these functions with $\pi_0 = 0$ and $\pi_1 = 1$. However, it has been found that by changing the initialization for $\pi_1$ to $\pi_1 = ½$ the post n-loop multiplications in Equation 22 can be avoided.

Algorithmic testing was carried out by comparison with special cases, limiting cases, and published results. The code was checked for internal consistency according to several *a priori*[5] criteria: no calculation ever yielded extinction and scattering efficiencies that were negative, the extinction efficiency was always greater than the scattering efficiency except for nonabsorbing particles for which these efficiencies were found to be equal, and predictions of the asymptotic limits of the efficiencies for both very small and very large size parameters were verified. Published results were used to confirm the calculations for both efficiencies and angular scattering quantities. The tabulated values of Wiscombe,[10] Dave,[3] and Bohren and Huffman[5] were used to check the code.

The maximum size parameter for which a valid scattering calculation can be done is determined by the ability to compute $J_\nu(mX)$ (Equations 15 and 16). Scaled values of these Bessel functions are computed by means of a uniform asymptotic expansion for $X$, $\nu >> 1$ which is valid for

$$\nu \cdot \mathrm{Re}\left\{ \ln\left[\frac{1+\sqrt{1+z^2}}{z}\right] - \sqrt{1+z^2} + z \right\} > \delta, \tag{25}$$

where $z^2 = -\left(\frac{mX}{\nu}\right)^2$, $|mX| \le \sqrt{\Gamma}$, and $2 \cdot \Gamma$ = largest positive machine integer. The first two terms on the left in Equation 25 arise from the theory for the uniform asymptotic expansion of Bessel functions,[9] and the last term comes about because scaled functions are being computed. For large size parameters it is known[4] that $\nu \approx X$ which leads to the following approximations for estimating the largest size parameters for which valid scattering calculations can be performed:

$$X_{\max} = \frac{\delta}{\mathrm{Re}\left\{ \ln\left[\frac{1+\sqrt{1+m^2}}{i\,m}\right] - \sqrt{1+m^2} + im \right\}} \tag{26}$$

or

$$X_{\max} = \frac{\sqrt{\Gamma}}{|m|}$$

The Figure shows a plot of results computed from Equation 26 on an AT&T 3B2-600G computer, which had the machine constants: $\delta \approx 702$, $2 \cdot \Gamma = 2^{31} - 1$. The Table displays a comparison of directly computed size parameters with the approximate values at the "three corners" shown in the Figure, and the maximum and minimum size parameters.

Table. Size Parameter Comparisons[a]

| n | k | $X_{max}$ | $X_{min}$ | $X_{est}$ |
|---|---|---|---|---|
| 1.05 | 0.95 | 2697 | 2700 | 2841 |
| 1.05 | 5.00 | 6401 | 6421 | 6414 |
| 5.00 | 5.00 | 4614 | 4635 | 4634 |
| 5.00 | 0.10 | 6468 | 6899 | 6552 |
| 1.40 | 0.10 | 23098 | 23118 | 23346 |

[a]n and k are the real and imaginary parts respectively of the complex refractive index. $X_{max}$ is the largest size parameter for which a calculation was completed. $X_{min}$ is the smallest size parameter for which the calculation failed. $X_{est}$ is the estimate of $X_{max}$ from Equation 29. Size parameters for finding $X_{max}$ and $X_{min}$ were equally spaced logarithmically with 50 values per decade.

In general the approximation overestimates the size parameter for which valid calculations can be performed; the maximum overestimation based on the data shown is on the order of five percent.

## 4. RESULTS AND DISCUSSION

The listings of a main (driver) program (msmain), the magnetic sphere scattering program (magsph) and associated programs are given in Appendix A. For each

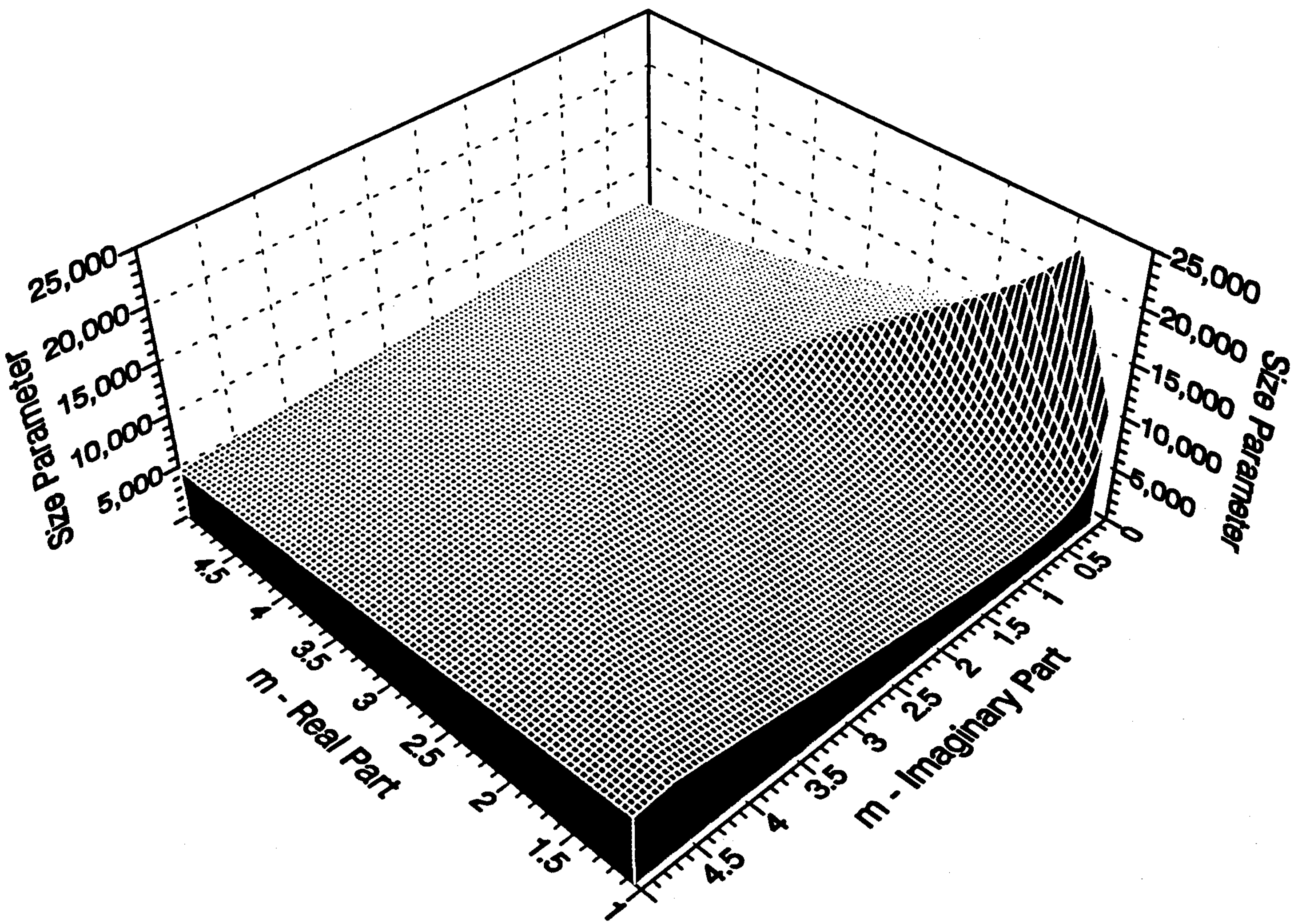


Figure. Maximum Size Parameter Estimates for Valid Scattering Calculations as a Function of the Real and Imaginary Parts of the Refractive Index

scattering angle the program computes the complex scattered field amplitudes ($S_1$, $S_2$), the scattered intensity

$$I = ½[\,|S_1|^2 + |S_2|^2\,], \tag{27}$$

the degree of polarization,

$$P = \frac{[\,|S_2|^2 - |S_1|^2\,]}{[\,|S_2|^2 + |S_1|^2\,]}, \tag{28}$$

the Mueller matrix elements ($S_{11}$, $S_{12}$, $S_{33}$, $S_{34}$) which are nonzero, independent quadratic combinations of $S_1$ and $S_2$ and elements of a 4 x 4 matrix relating the incident Stokes vector,[5]

$$\begin{aligned} S_{11} &= ½(\,|S_2|^2 + |S_1|^2\,) \\ S_{12} &= ½(\,|S_2|^2 - |S_1|^2\,) \\ S_{33} &= ½(S_2^* S_1 + S_2 S_1^*) \\ S_{34} &= ½(S_2^* S_1 - S_2 S_1^*) \end{aligned} \tag{29}$$

and the polarization

$$p = -S_{12}/S_{11}. \tag{30}$$

Efficiency factors (extinction, scattering, absorption, and backscatter) and the asymmetry factor,[5] g, (<cos$\theta$>) given by,

$$g = \frac{4}{X^2 Q_s} \sum_{n=1}^{\infty} \frac{n(n+2)}{n+1} \mathrm{Re}[a_n a_n^* + b_n b_{n+1}^*] + \frac{2n+1}{n(n+1)} \mathrm{Re}[a_n b_n^*] \tag{31}$$

are also computed. If the sphere is small, a comparison is made between the rigorous magsph calculation and an approximate calculation valid for small spheres (msphsx).

Appendix B lists input data to msmain and then displays the resulting output for two sample calculations. The first sample problem computes the scattering from spheres with a refractive index of (1.5, -.1), a permeability of (1.0, 0.), and size parameters of 0.01

and 10.0. The results from the latter size parameter may be compared with the published results of Wiscombe.[10] The second sample problem computes the scattering from a magnetic sphere with $\tilde{\mu} = \tilde{\epsilon} = (2.24, -.3)$ and the same size parameters as above. As noted by Kerker[2] the backscatter from a sphere with the same permeability and permittivity vanishes; the computed results illustrate this.

Blank

## LITERATURE CITED

1. Mie, G., "Beitrage zur Optik trüber Medien speziell kolloidaler Metallösungen," Ann. Phys. Vol. 25, pp 377-445 (1908).

2. Kerker, M., Wang, D.-S., and Giles, C.L., "Electromagnetic Scattering by Magnetic Spheres," J. Opt. Soc. Am. Vol. 73, pp 765-767 (1983).

3. Dave, J.V., "Subroutines for Computing the Parameters of the Electromagnetic Radiation Scattered by a Sphere," Rep. No. 320-3237, IBM Scientific Center, Palo Alto, CA (1968).

4. Wiscombe, W.J., "Improved Mie Scattering Algorithms," Appl. Opt. Vol. 19, pp 1505-1509 (1980).

5. Bohren, C.F., and Huffman, D.R., Absorption and Scattering of Light by Small Particles, Wiley, New York, NY, 1983.

6. Barber, P.W., and Hill, S.C., Light Scattering by Particles: Computational Methods, World Scientific, Teaneck, NJ, 1990.

7. Amos, D.E., "Algorithm 644: A Portable Package for Bessel Functions of a Complex Argument and Nonnegative Order," ACM Trans Math Software Vol. 12, pp 265-273 (1986).

8. Van de Hulst, H.C., Light Scattering by Small Particles, Dover, New York, NY, 1981.

9. Abramowitz, M., and Stegun, I.A., Handbook of Mathematical Functions, GPO, Washington, DC, 1964.

10. Wiscombe, W.J., "Mie Scattering Calculations: Advances in Technique and Fast, Vector-Speed Computer Codes," NCAR/TN-140+STR, National Center for Atmospheric Research, Boulder, CO, 1979.

11. Khare, V., Short-Wavelength Scattering of Electromagnetic Waves by a Homogenous Dielectric Sphere, Ph.D. Thesis, U. Rochester, NY, 1976.

12. Infield, L., "The Influence of the Width of the Gap Upon the Theory of Antennas," Quart. Appl. Math. Vol. 5, pp 113-132 (1947).

Blank

# APPENDIX A

## PROGRAM LISTINGS

```
      program msmain
c***************************************************************************
c
c   Program msmain computes the electromagnetic scattering for magnetic
c   spheres.  If the sphere is small a comparison is made between the
c   results of a rigorous, complete calculation and an approximate
c   calculation valid for small spheres.
c
c         Merrill Milham        >>> version 1.0 <<<           JANUARY 1994
c
c   inputs:
c                          ALL INPUT IS IN LIST DIRECTED FORMAT
c
c          line #1
c                          flag = 'r' for refractive index data or
c                                 'p' for permittivity data        (character*1)
c                          mior = number of complex indexes or
c                                 complex permittivities to be read    (integer)
c                            mp = complex index or permittivity values
c                                 to be read, up to 10 values      (complex*16)
c          line #2
c                           muu = complex permeability values to be read
c                                 mior values are required         (complex*16)
c          line #3
c                            nx = number of size parameters to be read (integer)
c                             x = size parameter values to be read     (real*8)
c          line #4  anginc = angular increment in degrees which is
c                                 added to zero to produce the angles at
c                                 which the scattering is to be produced.
c                                 Values of anginc are restricted to those
c                                 for which mod(90,anginc).eq.zero.     (real*8)
c
c   output:
c          For each angle the following quantities are given:
c                             The complex scattered field amplitudes, the
c                             scattered intensity, the degree of polarization,
c                             the Mueller matrix elements, and the polarization.
c
c          Efficiency factors for extinction, scattering, absorption, and
c          backscatter, and the asymmetry factor.
c
c   subroutines used:
c
c              magsph - returns computed scattering quantities for a
c                       homgeneous magnetic sphere or homogeneous
c                       nonmagnetic sphere if the permeability = (1,0)
c
c              amuelr - returns elements of the Mueller matrix for a
c                       spherical scatterer
c
c              msphsx - returns approximate scattering quantities for a
c                       small homogeneous, magnetic sphere or a small,
```

```
c                          homogeneous, nonmagnetic sphere if the
c                          permeability = (1,0)
c
c*****************************************************************************
c
      implicit none
*
      integer nangl
      real*8 one,zero
      parameter (nangl=255,one=1.d0,zero=0.d0)
*
      complex*16  s1(nangl), s2(nangl),sx1(nangl),sx2(nangl)
      complex*16 eps,mu,muu(10),mp(10)
*
      real*8 i1,i2,inten,theta(nangl),xx,x(10),anginc
      real*8 isx1,isx2,intsx,dgplsx,angle,degpol
      real*8 s11(nangl),s12(nangl),s33(nangl)
      real*8 s34(nangl),s11nor,pol(nangl)
      real*8 sx11(nangl),sx12(nangl),sx33(nangl)
      real*8 sx34(nangl),sxpol(nangl)
      real*8 qext,qsca,qbac,g,qabs
      real*8 qesx,qssx,qasx,qbsx
*
      integer sign,mior,i,nx,nmangl,nmang2,nior,nxx
*
      logical xflag
*
      character*1 flag
c
      write(*,*) 'read(*,*) flag,mior,(mp(i),i=1,mior)'
      read(*,*) flag,mior,(mp(i),i=1,mior)
      write(*,*) 'read(*,*) (muu(i),i=1,mior)'
      read(*,*) (muu(i),i=1,mior)
      write(*,*)'read(*,*) nx,(x(i),i=1,nx)'
      read(*,*) nx,(x(i),i=1,nx)
      write(*,*) 'read(*,*) anginc'
      read(*,*) anginc
      write(*,*)
c
      if(dmod(90.d0,anginc).eq.zero) then
         continue
                                        else
         stop 'angular increment error'
      end if
*
      nmangl=(180.d0/anginc)+1
      nmang2=(90.d0/anginc)+1
*
      do 1 i = 1,nmangl
    1 theta(i) = dble(i-1)*anginc
c
      do 100 nior = 1,mior
```

```
      if(flag.eq.'p') then
         eps=mp(nior)
        else if(flag.eq.'r') then
               eps=(mp(nior))**2
           else
             stop 'material properties input error'
      end if

      mu=muu(nior)
*
      sign=1
c
      do 100 nxx = 1,nx
      xx=x(nxx)
      xflag=xx*zabs(zsqrt(mu*eps)).lt.one
*
      write(*,1008) xx,eps,mu
      if(flag.eq.'r') write(*,1000) xx,mp(nior)
c
      call magsph(xx,eps,mu,nmang2,theta,qext,qsca,qbac,g,s1,s2)
      qabs = qext-qsca
*
      s11nor=0.d0
      call amuelr(s1,s2,theta,nmang1,sign,s11nor,s11,s12,s33,s34,pol)
c
      if(xflag) then
      call msphsx(xx,mu,eps,nmang2,theta,qesx,qssx,qasx,qbsx,sx1,sx2)
      s11nor=0.d0
      call amuelr(sx1,sx2,theta,nmang1,sign,s11nor,sx11,sx12,sx33,sx34
     &                                  ,sxpol)
                    else
      continue
      end if
c
      do 10 i = 1,nmang1
      angle = theta(i)
*
      i1 = (dble(s1(i)))**2+(dimag(s1(i)))**2
      i2 = (dble(s2(i)))**2+(dimag(s2(i)))**2
      inten = 0.5*(i1+i2)
      if(.not.(i1.eq.zero.and.i2.eq.zero)) then
      degpol = (i2-i1)/(i2+i1)
                                               else
      write(*,*)
      write(*,*) 'degree of polarization undefined for ',angle,' deg'
      write(*,*)
      degpol=zero
      end if
*
      if(xflag) then
      isx1 = (dble(sx1(i)))**2+(dimag(sx1(i)))**2
      isx2 = (dble(sx2(i)))**2+(dimag(sx2(i)))**2
```

```
      intsx = 0.5*(isx1+isx2)
      if(.not.(isx1.eq.zero.and.isx2.eq.zero)) then
      dgplsx = (isx2-isx1)/(isx2+isx1)
                                                  else
      write(*,*)
      write(*,*)'degree of polarization undefined for',angle,' deg.(sx)'
      write(*,*)
      dgplsx=zero
      end if
                          else
      continue
      end if
*
      write(*,1004)
      write(*,1001) angle,s1(i),s2(i)
      if(xflag) write(*,1001) angle,sx1(i),sx2(i)
      write(*,*)
      write(*,1005)
      write(*,1006) inten,degpol
      if(xflag) write(*,1006) intsx,dgplsx
      write(*,*)
      write(*,1007)
      write(*,1003) s11(i),s12(i),s33(i),s34(i),pol(i)
      if(xflag) write(*,1003) sx11(i),sx12(i),sx33(i),sx34(i),sxpol(i)
   10 write(*,*)
      write(*,1002) qext,qsca,qabs,g,qbac
      if(xflag) write(*,1002) qesx,qssx,qasx,g,qbsx
      write(*,*)
  100 continue
c
      stop
c
 1000 format(1x,'mie size parameter =',f10.5,5x,'refractive index ='
     +   ,f7.3,e12.3//)
 1001 format(f7.2,4e14.6)
 1002 format( /29x,'extinction    scattering    absorption'/
     +   7x,'efficiency factors',3e14.6/ 7x,'asymmetry factor =',f9.6,
     &   7x,'backscatter =',e14.6)
 1003 format(5e14.6)
 1004 format ('  angle',11x,'s-sub-1',21x,'s-sub-2')
 1005 format (11x,'intensity',4x,'deg of polzn')
 1006 format(7x,2e14.6)
 1007 format(7x,'s11',11x,'s12',11x,'s33',11x,'s34',11x,'pol')
 1008 format(1x,'mie size parameter =',f10.5,5x,'permittivity ='
     +   ,2e12.3,/36x,'permeability =',2e12.3//)
c
      end
```

```
      subroutine magsph(x,eps,mu,numang,theta,
     &                                        qext,qsca,qbac,g,s1,s2)
c
c ************************************************************************
c
c Subroutine magsph computes the scattering cross sections and angular
c scattering from a magnetic sphere. If the number of scattering angles
c is set to zero, only the cross sections (efficiencies) are returned.
c
c    Merrill Milham            >>> version 2.0 <<<            SEPT 1993
c
c    Inputs:
c             x = size parameter of the sphere                (real*8)
c           eps = complex permittivity: epsr -i*epsi          (complex*16)
c            mu = complex permeability: mur - i*mui           (complex*16)
c        numang = number of scattering angles                 (integer)
c                 between 0 & 90 deg.
c         theta = scattering angles in degrees                (real*8)
c              theta(i) are entered between 0 & 90 deg.
c              theta must increase monotonically. Results for
c              supplementary angles (180 deg. - theta(i)) are
c              also returned.
c
c    Outputs:
c          qext = extinction efficiency                       (real*8)
c          qsca = scattering efficiency                       (real*8)
c          qbac = backscatter efficiency                      (real*8)
c             g = asymmetry factor                            (real*8)
c            s1 = scattered amplitude                         (complex*16)
c            s2 = scattered amplitude                         (complex*16)
c
c    Subroutines used:
c
c           zbjy - returns one-half integer order J & Y Bessel functions
c
c    References:
c
c    M. Kerker, D.-S. Wang, and C. L. Giles, "Electromagnetic scattering
c    by magnetic spheres," J. Opt. Soc. Am., 73, 765-767 (1983).
c
c    D. E. Amos, "Algorithm 644:A portable package for Bessel functions of

c    a complex argument and nonnegative order," ACM Trans. on Math.
c    Software, 12, 265-273 (1986).
c
c    M. Abramowitz and I. A. Stegun, "Handbook of Mathematical Functions,"

c    NBS Applied Math. Series 55, US Dept. of Commerce, Washington, DC
c    (1955).
c
c    W. J. Wiscombe,"Mie Scattering Calculations: Advances in Technique
```

```
and
c     Fast, Vector-Speed Computer Codes," NCAR Tech. Note, NCAR/TN-140+STR
c     (1979)
c
c
****************************************************************************
c
      implicit none
c
      real*8 x
      complex*16 eps,mu
      integer numang
      real*8 theta(1)
*
      real*8 qext,qsca,qbac,g
      complex*16 s1(1),s2(1)
c
      integer al,nangl,nangl2
      real*8 third
      parameter (third=1.d0/3.d0,al=5100,nangl=255,nangl2=(nangl+1)/2)
      complex*16 sp(nangl2),sm(nangl2),sps(nangl2),sms(nangl2)
*
      complex*16 m,mc1,mxi,s,t,u,v,an,bn,xp
      real*8 xi,dn,dnn,rn,tnp1,thetan
      real*8 xmu(nangl),pi(nangl),pil(nangl),tau(nangl)
*
      real*8 bjr(al),byr(al)
      real*8 cjr(al),cji(al),cyr(al),cyi(al)
      real*8 bjn,byn,bj1
      real*8 sc,ca,t1,t2,t3,t4
      complex*16 cjn,cyn,cj1,cy1,h2n,h21
      complex*16 an1,bn1,bs,anp,bnp,abp,abm,anpm,bnpm
      complex*16 zt1,zt2
*
      integer kstop,k,mm,n,j,j2,j3
*
      real*8 cpi,zero,one,two,rad,half,fnu
      complex*16 cdblei,cdble1,cdble0
      parameter (cpi=3.14159265358979323846d0,zero=0.d0,one=1.d0)
      parameter (two=2.d0,rad=cpi/180.d0,half=0.5d0,fnu=half)
      parameter (cdblei=(0.d0,1.d0),cdble1=(1.d0,0.d0))
      parameter (cdble0=(0.d0,0.d0))
c
      kstop=idint(x+4.d0*x**third+4.d0)
*
      if(kstop.le.al) then
         continue
                     else
         print*,'magsph arrays too small: kstop =',kstop,' al =',al
         stop
      end if
c
```

```
      if(numang.eq.0) then
         s1(1)=cdble0
         s2(1)=cdble0
                     else
*
            if(numang.le.nangl2) then
               continue
                                 else
         print*,numang,'scattering angles input: only',nangl2,' allowed'
               stop
            endif
*
      do 100 n=1,numang
      thetan=dabs(theta(n))
      theta(n)=thetan
*
            if(thetan.le.90.d0) then
               continue
                                else
      print*,'theta(',n,')=',thetan,'scattering angles must be < 90 deg'
            stop
            end if
*
      thetan=rad*thetan
      xmu(n)=dcos(thetan)
*
      sp(n)=cdble0
      sm(n)=cdble0
      sps(n)=cdble0
      sms(n)=cdble0
      pi(n)=half
      pil(n)=zero
*
  100 continue
*
      end if
c
      m=zsqrt(mu*eps)
      mc1=m/mu
*
      xi=one/x
      mxi=xi/m
c
      call zbjy(x,m,kstop,fnu,bjr,byr,cjr,cji)
c
      bj1=bjr(1)
      cj1=dcmplx(cjr(1),cji(1))
      cy1=dcmplx(cyr(1),cyi(1))
      h21=dcmplx(bjr(1),-byr(1))
*
      qext=zero
      qsca=zero
```

```
      bs=cdble0
      g=zero
*
      anl=cdble0
      bnl=cdble0
      dn=one
      rn=one
      tnp1=one
      mm=1
c
      do 300 k=2,kstop
*
      tnp1=tnp1+two
      t1=dn-rn
      ca=one+rn
      sc=rn
*
      dnn=dn+one
      rn=one/dnn
*
      sc=sc+rn
*
      bjn=bjr(k)
      byn=byr(k)
      cjn=dcmplx(cjr(k),cji(k))
      cyn=dcmplx(cyr(k),cyi(k))
      h2n=dcmplx(bjn,-byn)
*
      xp=dn*mxi
      s=cjl-xp*cjn
      u=s*h2n
      s=s*bjn
*
      xp=dn*dcmplx(xi,zero)
      t=cjn*(bjl-xp*bjn)
      v=cjn*(h2l-xp*h2n)
*
      an=(s-mcl*t)/(u-mcl*v)
      bn=(mcl*s-t)/(mcl*u-v)
      abp=an+bn
      abm=an-bn
*
      zt1=dconjg(an)
      zt2=dconjg(bn)
      qext=qext+tnp1*dble(abp)
      qsca=qsca+tnp1*(an*zt1+bn*zt2)
      bs=bs-(dn+half)*mm*abm
      g=g+t1*dble(anl*zt1+bnl*zt2)+sc*dble(an*zt2)
*
      if(numang.eq.0) then
         continue
                       else
```

```
         anp=sc*abp
         bnp=sc*abm
         anpm=mm*anp
         bnpm=mm*bnp
*
         do 375 j=1,numang
         t1=xmu(j)*pi(j)
         t4=t1-pi1(j)
         tau(j)=dn*t4-pi1(j)
         t2=pi(j)+tau(j)
         t3=pi(j)-tau(j)
*
         sp(j)=sp(j)+anp*t2
         sms(j)=sms(j)+bnpm*t2
         sm(j)=sm(j)+bnp*t3
         sps(j)=sps(j)+anpm*t3
*
         pi1(j)=pi(j)
         pi(j)=t1+ca*t4
*
  375    continue
      end if
*
      dn=dnn
      mm=-mm
      an1=an
      bn1=bn
*
      bj1=bjn
      cj1=cjn
      cy1=cyn
      h21=h2n
*
  300 continue
c
      if(numang.eq.0) then
         continue
                     else
         j2=2*numang
         do 500 j=1,numang
         j3=j2-j
         s1(j)=sp(j)+sm(j)
         s2(j)=sp(j)-sm(j)
         s1(j3)=sps(j)+sms(j)
         s2(j3)=sps(j)-sms(j)
  500    continue
*
      end if
c
      xi=two*xi*xi
      qext=xi*qext
      qsca=xi*qsca
```

```
      xi=two*xi
      qbac=xi*bs*dconjg(bs)
      g=xi/qsca*g
c
      return
c
      end
```

```
      subroutine zbjy(x,m,nstop,fnu,
     &                             bjr,byr,cjr,cji)
c
c
*******************************************************************************
c
c  Subroutine zbjy gets J & Y Bessel functions for use in
c  sphere (fnu=0.50) or cylinder (fnu=0.0d0) scattering calculations.
c  Scaled or nonscaled functions for argument z = m*x are returned
depending
c  on the magnitude of the product of the size parameter and the complex
c  refractive index (zabs(z)).  Nonscaled functions are returned if the
c  imaginary part of the refractive index is zero.  Nonscaled functions are

c  returned for argument x.
c
c           Merrill Milham     >>> version: 2.0 <<<             JANUARY 1994
c
c  inputs:
c               x = the size parameter of the cylinder.(real*8)
c               m = the complex refractive index, n - ik.(complex*16)
c           nstop = the highest order of the Bessel functions.(integer)
c             fnu = 0.5d0 for sphere calculations or
c                   0.0d0 for cylinder calculations. (real*8)
c
c  outputs:
c
c   bjr = real part of J(x) Bessel functions.(array: real*8)
c   byr = real part of Y(x) Bessel functions.(array: real*8)
c   cjr = real part of J(m*x) Bessel functions.(array: real*8)
c   cji = imag. part of J(m*x) Bessel functions.(array: real*8)
c
c  subroutines used:
c
c                zbesj - returns J Bessel functions
c                zbesy - returns Y Bessel functions
c
c Reference: D. E. Amos, "Algorthim 644: A Portable Package for Bessel
c            Functions of a Complex Argument and Nonnegative Order,"
c            ACM Transcations on Mathematical Software, 12,265-273(1986).
c
c*******************************************************************************
**
c
      implicit none
      automatic cwrkr,cwrki,bji,byi
*
      integer al
      real*8 zero,xll,two
      parameter (al=5100,zero=0.0d0,xll=1.d-1,two=2.d0)
*
      real*8 bjr(1),byr(1)
```

```
      real*8 x,fnu,cjr(1),cji(1)
      real*8 zr,zi,d1mach
      real*8 cwrkr(al),cwrki(al)
      real*8 r1m5,elim,aa,alim
      complex*16 m,z
      integer kode,ierr,nz,nstop,n
      integer k1,i1mach,k2,k
      logical zflag,eflg1,eflg2
c
      kode=1
c
      call zbesj(x,zero,fnu,kode,nstop,bjr,cji,nz,ierr)
*
      eflg1=ierr.eq.0
      eflg2=nz.eq.0
      if (eflg1.and.eflg2) then
          continue
         else if (.not.eflg2.and.eflg1) then
                  nstop=nstop-nz
                  print*,'zbesj error: ierr =',ierr,'nz =',nz
                    else
              print*,'inputs =',x,zero,fnu,kode,nstop
              print*,'zbesj called from subroutine zbjy'
      end if
c
      call zbesy(x,zero,fnu,kode,nstop,byr,cji,nz,cwrkr,cwrki,ierr)
*
      eflg1=ierr.eq.0
      eflg2=nz.eq.0
      if (eflg1.and.eflg2) then
          continue
         else if (.not.eflg2.and.eflg1) then
                  nstop=nstop-nz
                  print*,'zbesy error: ierr =',ierr,'nz =',nz
          continue
                    else
              print*,'zbesy error: ierr =',ierr,'nz =',nz
              print*,'inputs =',x,zero,fnu,kode,nstop
              print*,'zbesy called from subroutine zbjy'
      end if
c
      z=m*x
      zr=dble(z)
      zi=dimag(z)
      if(zi.ne.zero) then
*
      k1 = i1mach(15)
      k2 = i1mach(16)
      r1m5 = d1mach(5)
      k = min0(iabs(k1),iabs(k2))
      elim = 2.303d0*(dble(float(k))*r1m5-3.0d0)
      k1 = i1mach(14) - 1
```

```
      aa = r1m5*dble(float(k1))
      aa = aa*2.303d0
      alim = elim + dmax1(-aa,-41.45d0)
*
         if(dabs(zi).gt.alim) kode=2
                                          else
         continue
      end if
c
      call zbesj(zr,zi,fnu,kode,nstop,cjr,cji,nz,ierr)
*
      eflg1=ierr.eq.0
      eflg2=nz.eq.0
      if (eflg1.and.eflg2) then
          continue
         else if (.not.eflg2.and.eflg1) then
                  nstop=nstop-nz
                  print*,'zbesj error: ierr =',ierr,'nz =',nz
          continue
                    else
             print*,'zbesj error: ierr =',ierr,'nz =',nz
             print*,'inputs =',zr,zi,fnu,kode,nstop
             print*,'zbesj called from subroutine zbjy'
      end if
c
      zflag=dabs(zi).lt.two*d1mach(1).and.x.lt.xll
      if (.not.zflag) then
           continue
                    else
           do 100 n=1,nstop
              cji(n)=zero
  100      continue
      end if
*
      zflag=dabs(zr).lt.two*d1mach(1).and.x.lt.xll
      if (.not.zflag) then
           continue
                    else
           cji(1)=zero
           do 200 n=2,nstop
                  if(mod(n,2).eq.0) then
                     cjr(n)=zero
                                        else
                     cji(n)=zero
                  end if
  200      continue
      end if
c
      return
c
      end
```

```
      subroutine msphsx(x,mu,eps,ntheta,theta,
     &                                       qext,qsca,qabs,qbac,s1,s2)
c
c   ***********************************************************************
c
c   Subroutine msphsx computes an approximate values of the scattered amplitude:
c   and scattering cross sections (efficiencies) for a magnetic sphere with
c   small size parameters.
c
c      Merrill Milham            >>> version 1.2 <<<              DEC 1993
c
c      Inputs:
c               x = size parameter of the sphere      (real*8)
c             eps = complex permittivity: epsr -i*epsi (complex*16)
c              mu = complex permeability: mur - i*mui  (complex*16)
c          ntheta = number of scattering angles        (integer)
c           theta = scattering angles in degrees       (real*8)
c
c      Outputs:
c              s1 = scattered amplitude                (complex*16)
c              s2 = scattered amplitude                (complex*16)
c            qext = extinction efficiency              (real*8)
c            qsca = scattering efficiency              (real*8)
c            qabs = absorption efficiency              (real*8)
c            qbac = backscatter efficiency             (real*8)
c
c      References:
c
c      M. Kerker, D.-S. Wang, and C. L. Giles, "Electromagnetic scattering
c      by magnetic spheres," J. Opt. Soc. Am., 73, 765-767 (1983).
c
c      J. A. Stratton, "Electromagnetic Theory," (McGraw-Hill, New York,
c      1941)
c
c   ************************************************************************
c
      real*8 x
      complex*16 mu,eps
      integer ntheta
      real*8 theta(1)
*
      complex*16 s1(1),s2(1)
      real*8 qext,qsca,qabs,qbac
*
      real*8 xp,n,k,thetal
      complex*16 ef,mf,mui,epsi,cx
*
      real*8 zero,one,two,three,four,eight,c0,cpi,rad
      parameter (zero=0.d0,one=1.d0,two=2.d0,four=4.d0)
      parameter (three=3.d0,eight=8.d0,c0=eight/three)
      parameter (cpi=3.14159265358979323840d0,rad=cpi/180.d0)
      complex*16 cdble0,cdblei
```

```
      parameter (cdble0=(0.d0,0.d0),cdblei=(0.d0,1.d0))
*
      integer nangl,nangl2
      parameter (nangl=255,nangl2=(nangl+1)/2)
      real*8 xmu(nangl)
      integer l,j,j2,j3
c
      if(ntheta.eq.0) then
         s1(1)=cdble0
         s2(1)=cdble0
                    else
*
           if(ntheta.le.nangl2) then
              continue
                                    else
         print*,ntheta,'scattering angles input: only',nangl2,' allowed'
              stop
           endif
*
      do 100 l=1,ntheta
      thetal=dabs(theta(l))
      theta(l)=thetal
*
           if(thetal.le.90.d0) then
              continue
                                   else
      print*,'theta(',l,')=',thetal,'scattering angles must be <=90 deg'
           stop
           end if
*
      thetal=rad*thetal
      xmu(l)=dcos(thetal)
  100 continue
*
      end if
c
      n=dble(eps)
      k=dabs(dimag(eps))
      epsi=dcmplx(n,-k)
*
      n=dble(mu)
      k=dabs(dimag(mu))
      mui=dcmplx(n,-k)
c
      ef=(epsi-one)/(epsi+two)
      mf=(mui-one)/(mui+two)
*
      xp=x*x*x
      cx=dcmplx(zero,xp)
c
      if(ntheta.eq.0) then
         continue
```

```
                    else
          j2=2*ntheta
          do 500 j=1,ntheta
          j3=j2-j
          s1(j)=cx*(ef+mf*xmu(j))
          s2(j)=cx*(ef*xmu(j)+mf)
          s1(j3)=cx*(ef-mf*xmu(j))
          s2(j3)=cx*(mf-ef*xmu(j))
 500      continue
      end if
c
      xp=xp*x
      qext=-four*x*dimag(ef+mf)
      qsca=c0*xp*(zabs(ef)**2+zabs(mf)**2)
      if(qext.le.zero) qext=qsca
      qabs=ddim(qext,qsca)
      qbac=four*xp*(zabs(ef-mf))**2
c
      return
c
      end
```

```
      subroutine amuelr (s1,s2,theta,numang,sign,
     &                                      s11nor,s11,s12,s33,s34,pol)
c
c ***********************************************************************
c  subroutine amuelr computes Mueller matrix elements for either
c  spheres or infinite cylinders
c
c     Merrill Milham        >>> version: 2.0 <<<        JANUARY 1994
c
c  inputs:
c           s1 = amplitude scattering matrix element array.(complex*16)
c           s2 = amplitude scattering matrix element array.(complex*16)
c        theta = scattering angle array                      (real*8)
c       numang = the number of scattering angles, i.e., number of
c                elements in s1, s2.                          (integer)
c         sign = arbitrary sign (+1 or -1) used to adjust the sign of
c                s34 according to the users convention.     (integer)
c  outputs:
c
c       s11nor = s11 for a scattering angle of zero degrees, which
c                is used to normalize s11(theta)              (real*8)
c          s11 = Mueller matrix element 1,1                   (real*8)
c          s12 = Mueller matrix element 1,2                   (real*8)
c          s33 = Mueller matrix element 3,3                   (real*8)
c          s34 = Mueller matrix element 3,4                   (real*8)
c          pol = polarization = pol=-s12/s11                  (real*8)
c
c          s12,s33,s34 are normalized by s11(theta)
c
c subroutines used:  none
c
c ********************************************************************
c
      implicit none
*
      complex*16 s1(1),s2(1)
      real*8 theta(1),s11nor
      integer numang,sign
*
      real*8 s11(1),s12(1),s33(1),s34(1)
      real*8 pol(1)
*
      real*8 rs1,is1,rs2,is2,s11i
      real*8 ts11,ts12,half,zero,one
      parameter (half=0.5d0,zero=0.d0,one=1.d0)
      complex*16 ts
      integer i
c
      do 100 i=1,numang
      rs1=dble(s1(i))
      is1=dimag(s1(i))
      rs2=dble(s2(i))
```

```
      is2=dimag(s2(i))
*
      ts11=rs1*rs1+is1*is1
      ts12=rs2*rs2+is2*is2
      s11(i)=half*(ts11+ts12)
      s12(i)=half*(ts12-ts11)
*
      if(s11(i).ne.zero) then
          s11i=s11(i)
                              else
          s11i=one
          write(*,*)
          write(*,*) 'Unnormalized Mueller matrix elements for ',
     &                 theta(i),' deg.'
          write(*,*)
      end if
*
      pol(i)=-s12(i)/s11i
*
      ts=s2(i)*dconjg(s1(i))
      s33(i)=dble(ts)
      s33(i)=s33(i)/s11i
      s34(i)=sign*dimag(ts)
      s34(i)=s34(i)/s11i
*
      if(i.eq.1) s11nor=s11i
*
      s11(i)=s11i/s11nor
  100 continue
c
      return
c
      end
```

# APPENDIX B

# SAMPLE CALCULATIONS

## INPUT DATA

```
'r',1,(1.5,-.1)
(1,0)
2,0.01,10.
10
```

## OUTPUT

```
read(*,*) flag,mior,(mp(i),i=1,mior)
read(*,*) (muu(i),i=1,mior)
read(*,*) nx,(x(i),i=1,nx)
read(*,*) anginc

mie size parameter =   0.01000     permittivity =   0.224E+01   -0.300E+00
                                   permeability =   0.100E+01    0.000E+00


mie size parameter =   0.01000     refractive index =  1.500   -0.100E+00


 angle          s-sub-1                          s-sub-2
  0.00  0.498158E-07  0.295985E-06  0.498158E-07  0.295985E-06
  0.00  0.498129E-07  0.295977E-06  0.498129E-07  0.295977E-06

          intensity     deg of polzn
        0.900888E-13  0.000000E+00
        0.900839E-13  0.000000E+00

      s11           s12           s33           s34           pol
 0.100000E+01  0.000000E+00  0.100000E+01  0.000000E+00 -0.000000E+00
 0.100000E+01  0.000000E+00  0.100000E+01  0.000000E+00 -0.000000E+00

 angle          s-sub-1                          s-sub-2
 10.00  0.498158E-07  0.295985E-06  0.490590E-07  0.291488E-06
 10.00  0.498129E-07  0.295977E-06  0.490562E-07  0.291481E-06

          intensity     deg of polzn
        0.887305E-13 -0.153075E-01
        0.887257E-13 -0.153076E-01

      s11           s12           s33           s34           pol
 0.984923E+00 -0.135824E-14  0.999883E+00 -0.333333E-07  0.153075E-01
 0.984923E+00 -0.135818E-14  0.999883E+00  0.355640E-16  0.153076E-01

 angle          s-sub-1                          s-sub-2
 20.00  0.498157E-07  0.295985E-06  0.468115E-07  0.278135E-06
 20.00  0.498129E-07  0.295977E-06  0.468088E-07  0.278128E-06
```

```
      intensity    deg of polzn
    0.848194E-13 -0.621218E-01
    0.848150E-13 -0.621224E-01

    s11          s12           s33           s34           pol
0.941509E+00 -0.526913E-14  0.998069E+00 -0.135275E-06  0.621218E-01
0.941511E+00 -0.526891E-14  0.998069E+00 -0.186019E-16  0.621224E-01

angle           s-sub-1                        s-sub-2
30.00  0.498156E-07  0.295984E-06  0.431417E-07  0.256330E-06
30.00  0.498129E-07  0.295977E-06  0.431393E-07  0.256324E-06

      intensity    deg of polzn
    0.788273E-13 -0.142856E+00
    0.788234E-13 -0.142857E+00

    s11          s12           s33           s34           pol
0.874995E+00 -0.112609E-13  0.989744E+00 -0.311081E-06  0.142856E+00
0.875000E+00 -0.112605E-13  0.989743E+00 -0.400318E-16  0.142857E+00

angle           s-sub-1                        s-sub-2
40.00  0.498155E-07  0.295984E-06  0.381611E-07  0.226737E-06
40.00  0.498129E-07  0.295977E-06  0.381589E-07  0.226732E-06

      intensity    deg of polzn
    0.714769E-13 -0.260377E+00
    0.714737E-13 -0.260379E+00

    s11          s12           s33           s34           pol
0.793405E+00 -0.186109E-13  0.965507E+00 -0.566993E-06  0.260377E+00
0.793412E+00 -0.186103E-13  0.965506E+00  0.220742E-16  0.260379E+00

angle           s-sub-1                        s-sub-2
50.00  0.498153E-07  0.295983E-06  0.320210E-07  0.190255E-06
50.00  0.498129E-07  0.295977E-06  0.320191E-07  0.190251E-06

      intensity    deg of polzn
    0.636548E-13 -0.415248E+00
    0.636522E-13 -0.415252E+00

    s11          s12           s33           s34           pol
0.706578E+00 -0.264325E-13  0.909708E+00 -0.904240E-06  0.415248E+00
0.706588E+00 -0.264317E-13  0.909706E+00  0.000000E+00  0.415252E+00

angle           s-sub-1                        s-sub-2
60.00  0.498151E-07  0.295982E-06  0.249080E-07  0.147992E-06
60.00  0.498129E-07  0.295977E-06  0.249065E-07  0.147989E-06

      intensity    deg of polzn
    0.563043E-13 -0.599995E+00
    0.563024E-13 -0.600000E+00
```

| s11 | s12 | s33 | s34 | pol |
|---|---|---|---|---|
| 0.624987E+00 | -0.337823E-13 | 0.800003E+00 | -0.130654E-05 | 0.599995E+00 |
| 0.625000E+00 | -0.337815E-13 | 0.800000E+00 | -0.280223E-16 | 0.600000E+00 |

| angle | s-sub-1 | | s-sub-2 | |
|---|---|---|---|---|
| 70.00 | 0.498148E-07 | 0.295981E-06 | 0.170382E-07 | 0.101232E-06 |
| 70.00 | 0.498129E-07 | 0.295977E-06 | 0.170370E-07 | 0.101230E-06 |

| intensity | deg of polzn |
|---|---|
| 0.503122E-13 | -0.790541E+00 |
| 0.503109E-13 | -0.790546E+00 |

| s11 | s12 | s33 | s34 | pol |
|---|---|---|---|---|
| 0.558473E+00 | -0.397738E-13 | 0.612409E+00 | -0.172148E-05 | 0.790541E+00 |
| 0.558489E+00 | -0.397730E-13 | 0.612403E+00 | 0.000000E+00 | 0.790546E+00 |

| angle | s-sub-1 | | s-sub-2 | |
|---|---|---|---|---|
| 80.00 | 0.498146E-07 | 0.295979E-06 | 0.865075E-08 | 0.513976E-07 |
| 80.00 | 0.498129E-07 | 0.295977E-06 | 0.864992E-08 | 0.513959E-07 |

| intensity | deg of polzn |
|---|---|
| 0.464009E-13 | -0.941455E+00 |
| 0.464001E-13 | -0.941458E+00 |

| s11 | s12 | s33 | s34 | pol |
|---|---|---|---|---|
| 0.515058E+00 | -0.436844E-13 | 0.337139E+00 | -0.205011E-05 | 0.941455E+00 |
| 0.515077E+00 | -0.436838E-13 | 0.337131E+00 | 0.000000E+00 | 0.941458E+00 |

| angle | s-sub-1 | | s-sub-2 | |
|---|---|---|---|---|
| 90.00 | 0.498143E-07 | 0.295978E-06 | 0.556044E-12 | 0.133481E-11 |
| 90.00 | 0.498129E-07 | 0.295977E-06 | 0.000000E+00 | 0.000000E+00 |

| intensity | deg of polzn |
|---|---|
| 0.450423E-13 | -0.100000E+01 |
| 0.450420E-13 | -0.100000E+01 |

| s11 | s12 | s33 | s34 | pol |
|---|---|---|---|---|
| 0.499976E+00 | -0.450423E-13 | 0.938616E-05 | -0.217760E-05 | 0.100000E+01 |
| 0.500000E+00 | -0.450420E-13 | 0.000000E+00 | 0.000000E+00 | 0.100000E+01 |

| angle | s-sub-1 | | s-sub-2 | |
|---|---|---|---|---|
| 100.00 | 0.498141E-07 | 0.295977E-06 | -0.864959E-08 | -0.513946E-07 |
| 100.00 | 0.498129E-07 | 0.295977E-06 | -0.864992E-08 | -0.513959E-07 |

| intensity | deg of polzn |
|---|---|
| 0.464000E-13 | -0.941461E+00 |
| 0.464001E-13 | -0.941458E+00 |

| s11 | s12 | s33 | s34 | pol |
|---|---|---|---|---|
| 0.515048E+00 | -0.436838E-13 | -0.337122E+00 | -0.205013E-05 | 0.941461E+00 |
| 0.515077E+00 | -0.436838E-13 | -0.337131E+00 | 0.000000E+00 | 0.941458E+00 |

```
 angle          s-sub-1                          s-sub-2
110.00  0.498139E-07  0.295976E-06 -0.170368E-07 -0.101229E-06
110.00  0.498129E-07  0.295977E-06 -0.170370E-07 -0.101230E-06

          intensity    deg of polzn
        0.503103E-13 -0.790550E+00
        0.503109E-13 -0.790546E+00

     s11           s12           s33           s34           pol
 0.558452E+00 -0.397728E-13 -0.612397E+00 -0.172151E-05  0.790550E+00
 0.558489E+00 -0.397730E-13 -0.612403E+00  0.000000E+00  0.790546E+00

 angle          s-sub-1                          s-sub-2
120.00  0.498136E-07  0.295975E-06 -0.249064E-07 -0.147986E-06
120.00  0.498129E-07  0.295977E-06 -0.249065E-07 -0.147989E-06

          intensity    deg of polzn
        0.563014E-13 -0.600005E+00
        0.563024E-13 -0.600000E+00

     s11           s12           s33           s34           pol
 0.624954E+00 -0.337811E-13 -0.799997E+00 -0.130658E-05  0.600005E+00
 0.625000E+00 -0.337815E-13 -0.800000E+00  0.280223E-16  0.600000E+00

 angle          s-sub-1                          s-sub-2
130.00  0.498134E-07  0.295974E-06 -0.320191E-07 -0.190247E-06
130.00  0.498129E-07  0.295977E-06 -0.320191E-07 -0.190251E-06

          intensity    deg of polzn
        0.636506E-13 -0.415256E+00
        0.636522E-13 -0.415252E+00

     s11           s12           s33           s34           pol
 0.706531E+00 -0.264313E-13 -0.909705E+00 -0.904265E-06  0.415256E+00
 0.706588E+00 -0.264317E-13 -0.909706E+00  0.000000E+00  0.415252E+00

 angle          s-sub-1                          s-sub-2
140.00  0.498132E-07  0.295973E-06 -0.381589E-07 -0.226728E-06
140.00  0.498129E-07  0.295977E-06 -0.381589E-07 -0.226732E-06

          intensity    deg of polzn
        0.714715E-13 -0.260382E+00
        0.714737E-13 -0.260379E+00

     s11           s12           s33           s34           pol
 0.793344E+00 -0.186098E-13 -0.965506E+00 -0.567010E-06  0.260382E+00
 0.793412E+00 -0.186103E-13 -0.965506E+00 -0.220742E-16  0.260379E+00

 angle          s-sub-1                          s-sub-2
150.00  0.498131E-07  0.295972E-06 -0.431393E-07 -0.256319E-06
150.00  0.498129E-07  0.295977E-06 -0.431393E-07 -0.256324E-06
```

| | intensity | deg of polzn |
|---|---|---|
| | 0.788207E-13 | -0.142858E+00 |
| | 0.788234E-13 | -0.142857E+00 |

| s11 | s12 | s33 | s34 | pol |
|---|---|---|---|---|
| 0.874922E+00 | -0.112602E-13 | -0.989743E+00 | -0.311091E-06 | 0.142858E+00 |
| 0.875000E+00 | -0.112605E-13 | -0.989743E+00 | 0.400318E-16 | 0.142857E+00 |

| angle | s-sub-1 | | s-sub-2 | |
|---|---|---|---|---|
| 160.00 | 0.498130E-07 | 0.295972E-06 | -0.468088E-07 | -0.278122E-06 |
| 160.00 | 0.498129E-07 | 0.295977E-06 | -0.468088E-07 | -0.278128E-06 |

| | intensity | deg of polzn |
|---|---|---|
| | 0.848118E-13 | -0.621229E-01 |
| | 0.848150E-13 | -0.621224E-01 |

| s11 | s12 | s33 | s34 | pol |
|---|---|---|---|---|
| 0.941424E+00 | -0.526876E-14 | -0.998069E+00 | -0.135280E-06 | 0.621229E-01 |
| 0.941511E+00 | -0.526891E-14 | -0.998069E+00 | 0.186019E-16 | 0.621224E-01 |

| angle | s-sub-1 | | s-sub-2 | |
|---|---|---|---|---|
| 170.00 | 0.498129E-07 | 0.295971E-06 | -0.490561E-07 | -0.291475E-06 |
| 170.00 | 0.498129E-07 | 0.295977E-06 | -0.490562E-07 | -0.291481E-06 |

| | intensity | deg of polzn |
|---|---|---|
| | 0.887222E-13 | -0.153078E-01 |
| | 0.887257E-13 | -0.153076E-01 |

| s11 | s12 | s33 | s34 | pol |
|---|---|---|---|---|
| 0.984831E+00 | -0.135814E-14 | -0.999883E+00 | -0.333345E-07 | 0.153078E-01 |
| 0.984923E+00 | -0.135818E-14 | -0.999883E+00 | -0.355640E-16 | 0.153076E-01 |

| angle | s-sub-1 | | s-sub-2 | |
|---|---|---|---|---|
| 180.00 | 0.498129E-07 | 0.295971E-06 | -0.498129E-07 | -0.295971E-06 |
| 180.00 | 0.498129E-07 | 0.295977E-06 | -0.498129E-07 | -0.295977E-06 |

| | intensity | deg of polzn |
|---|---|---|
| | 0.900803E-13 | 0.000000E+00 |
| | 0.900839E-13 | 0.000000E+00 |

| s11 | s12 | s33 | s34 | pol |
|---|---|---|---|---|
| 0.999906E+00 | 0.000000E+00 | -0.100000E+01 | 0.000000E+00 | -0.000000E+00 |
| 0.100000E+01 | 0.000000E+00 | -0.100000E+01 | 0.000000E+00 | -0.000000E+00 |

| | extinction | scattering | absorption |
|---|---|---|---|
| efficiency factors | 0.199263E-02 | 0.240226E-08 | 0.199263E-02 |

asymmetry factor = 0.000020 backscatter = 0.360321E-08

| | extinction | scattering | absorption |
|---|---|---|---|
| efficiency factors | 0.199252E-02 | 0.240224E-08 | 0.199251E-02 |

asymmetry factor = 0.000020 backscatter = 0.360336E-08

```
mie size parameter =  10.00000     permittivity =   0.224E+01  -0.300E+00
                                   permeability =   0.100E+01   0.000E+00

mie size parameter =  10.00000     refractive index =  1.500  -0.100E+00

 angle           s-sub-1                          s-sub-2
  0.00  0.614948E+02 -0.317799E+01  0.614948E+02 -0.317799E+01

        intensity    deg of polzn
      0.379171E+04  0.000000E+00

     s11           s12           s33           s34           pol
 0.100000E+01  0.000000E+00  0.100000E+01  0.000000E+00 -0.000000E+00

 angle           s-sub-1                          s-sub-2
 10.00  0.377490E+02  0.936674E+00  0.374475E+02 -0.207314E+01

        intensity    deg of polzn
      0.141624E+04 -0.679779E-02

     s11           s12           s33           s34           pol
 0.373510E+00 -0.962731E+01  0.996770E+00 -0.800255E-01  0.679779E-02

 angle           s-sub-1                          s-sub-2
 20.00  0.181377E+01  0.410068E+01  0.215939E+01  0.280384E+00

        intensity    deg of polzn
      0.124235E+02 -0.618335E+00

     s11           s12           s33           s34           pol
 0.327648E-02 -0.768186E+01  0.407810E+00 -0.671828E+00  0.618335E+00

 angle           s-sub-1                          s-sub-2
 30.00 -0.579008E+01 -0.121935E+01 -0.442757E+01  0.132155E+00

        intensity    deg of polzn
      0.273164E+02 -0.281719E+00

     s11           s12           s33           s34           pol
 0.720424E-02 -0.769552E+01  0.932586E+00 -0.225651E+00  0.281719E+00

 angle           s-sub-1                          s-sub-2
 40.00  0.115404E+01 -0.407492E+01  0.740364E+00 -0.184894E+01

        intensity    deg of polzn
      0.109517E+02 -0.637801E+00

     s11           s12           s33           s34           pol
 0.288834E-02 -0.698503E+01  0.765967E+00  0.806435E-01  0.637801E+00
```

```
angle              s-sub-1                             s-sub-2
50.00  0.166733E+01  0.121208E+01 -0.309906E+00 -0.269918E+00

         intensity    deg of polzn
       0.220902E+01 -0.923542E+00

     s11           s12           s33           s34           pol
0.582592E-03 -0.204012E+01 -0.382016E+00 -0.336851E-01  0.923542E+00

angle              s-sub-1                             s-sub-2
60.00 -0.693755E+00  0.314952E+01 -0.281778E-01  0.159291E+01

         intensity    deg of polzn
       0.646946E+01 -0.607670E+00

     s11           s12           s33           s34           pol
0.170621E-02 -0.393130E+01  0.778496E+00 -0.157099E+00  0.607670E+00

angle              s-sub-1                             s-sub-2
70.00 -0.113609E+01 -0.117949E+01  0.125557E+01 -0.241202E+00

         intensity    deg of polzn
       0.215826E+01 -0.242616E+00

     s11           s12           s33           s34           pol
0.569207E-03 -0.523630E+00 -0.529101E+00  0.813136E+00  0.242616E+00

angle              s-sub-1                             s-sub-2
80.00 -0.273185E+00 -0.219728E+01 -0.830355E+00 -0.629697E+00

         intensity    deg of polzn
       0.299433E+01 -0.637312E+00

     s11           s12           s33           s34           pol
0.789705E-03 -0.190832E+01  0.537836E+00 -0.551875E+00  0.637312E+00

angle              s-sub-1                             s-sub-2
90.00  0.135105E+01  0.417250E+00 -0.102255E+01  0.791253E+00

         intensity    deg of polzn
       0.183556E+01 -0.892756E-01

     s11           s12           s33           s34           pol
0.484100E-03 -0.163871E+00 -0.572777E+00  0.814835E+00  0.892756E-01

angle              s-sub-1                             s-sub-2
100.00  0.117126E+01  0.118164E+01  0.658108E+00 -0.351174E+00

         intensity    deg of polzn
       0.166228E+01 -0.665261E+00

     s11           s12           s33           s34           pol
```

```
 0.438398E-03 -0.110585E+01  0.214077E+00 -0.715262E+00  0.665261E+00

 angle            s-sub-1                           s-sub-2
110.00 -0.857020E+00  0.821659E+00  0.178123E+00 -0.127272E+01

         intensity    deg of polzn
       0.153058E+01  0.790379E-01

     s11           s12           s33           s34           pol
 0.403666E-03  0.120974E+00 -0.782971E+00  0.617016E+00 -0.790379E-01

 angle            s-sub-1                           s-sub-2
120.00 -0.145257E+01  0.316204E+00  0.255067E+00  0.235420E+00

         intensity    deg of polzn
       0.116521E+01 -0.896600E+00

     s11           s12           s33           s34           pol
 0.307304E-03 -0.104472E+01 -0.254085E+00 -0.362697E+00  0.896600E+00

 angle            s-sub-1                           s-sub-2
130.00 -0.630302E+00 -0.824923E+00  0.128067E+01  0.877085E+00

         intensity    deg of polzn
       0.174359E+01  0.381861E+00

     s11           s12           s33           s34           pol
 0.459842E-03  0.665806E+00 -0.877924E+00  0.288845E+00 -0.381861E+00

 angle            s-sub-1                           s-sub-2
140.00 -0.154838E+00 -0.112897E+01  0.566968E-01  0.542568E+00

         intensity    deg of polzn
       0.798073E+00 -0.627109E+00

     s11           s12           s33           s34           pol
 0.210479E-03 -0.500479E+00 -0.778529E+00 -0.250616E-01  0.627109E+00

 angle            s-sub-1                           s-sub-2
150.00  0.205857E+00 -0.889334E+00 -0.919354E+00  0.994698E+00

         intensity    deg of polzn
       0.133396E+01  0.375326E+00

     s11           s12           s33           s34           pol
 0.351811E-03  0.500672E+00 -0.805025E+00 -0.459418E+00 -0.375326E+00

 angle            s-sub-1                           s-sub-2
160.00  0.812356E+00 -0.110065E+01  0.709831E-01  0.104108E+01

         intensity    deg of polzn
       0.148012E+01 -0.264325E+00
```

```
       s11             s12             s33             s34             pol
 0.390358E-03  -0.391233E+00  -0.735212E+00   0.624176E+00   0.264325E+00

angle             s-sub-1                            s-sub-2
170.00   0.131138E+01  -0.469165E+00  -0.399742E+00   0.187150E+00

             intensity     deg of polzn
         0.106733E+01  -0.817470E+00

       s11             s12             s33             s34             pol
 0.281490E-03  -0.872507E+00  -0.573413E+00   0.542291E-01   0.817470E+00

angle             s-sub-1                            s-sub-2
180.00   0.149343E+01   0.296366E+00  -0.149343E+01  -0.296366E+00

             intensity     deg of polzn
         0.231818E+01   0.000000E+00

       s11             s12             s33             s34             pol
 0.611381E-03   0.000000E+00  -0.100000E+01   0.000000E+00  -0.000000E+00


                               extinction     scattering     absorption
    efficiency factors   0.245979E+01   0.123514E+01   0.122465E+01
    asymmetry factor = 0.922350         backscatter =   0.927271E-01
```

INPUT DATA

```
'p',1,(2.24,-.3)
(2.24,-.3)
2,0.01,10.
10
```

OUTPUT

```
 read(*,*) flag,mior,(mp(i),i=1,mior)
 read(*,*) (muu(i),i=1,mior)
 read(*,*) nx,(x(i),i=1,nx)
 read(*,*) anginc

 mie size parameter =   0.01000     permittivity =   0.224E+01  -0.300E+00
                                    permeability =   0.224E+01  -0.300E+00


 Unnormalized Mueller matrix elements for    180.0000000000000     deg.

  angle           s-sub-1                         s-sub-2
   0.00  0.996372E-07  0.591982E-06  0.996372E-07  0.591982E-06
   0.00  0.996258E-07  0.591955E-06  0.996258E-07  0.591955E-06

           intensity    deg of polzn
         0.360371E-12  0.000000E+00
         0.360336E-12  0.000000E+00

      s11           s12           s33           s34           pol
  0.100000E+01  0.000000E+00  0.100000E+01  0.000000E+00 -0.000000E+00
  0.100000E+01  0.000000E+00  0.100000E+01  0.000000E+00 -0.000000E+00

  angle           s-sub-1                         s-sub-2
  10.00  0.988804E-07  0.587485E-06  0.988804E-07  0.587485E-06
  10.00  0.988691E-07  0.587458E-06  0.988691E-07  0.587458E-06

           intensity    deg of polzn
         0.354916E-12  0.000000E+00
         0.354882E-12  0.000000E+00

      s11           s12           s33           s34           pol
  0.984865E+00  0.000000E+00  0.100000E+01  0.000000E+00 -0.000000E+00
  0.984865E+00  0.000000E+00  0.100000E+01  0.000000E+00 -0.000000E+00

  angle           s-sub-1                         s-sub-2
  20.00  0.966327E-07  0.574131E-06  0.966327E-07  0.574131E-06
  20.00  0.966218E-07  0.574105E-06  0.966218E-07  0.574105E-06

           intensity    deg of polzn
         0.338964E-12  0.000000E+00
```

```
        0.338932E-12   0.000000E+00

     s11              s12              s33              s34              pol
0.940600E+00   0.000000E+00   0.100000E+01   0.000000E+00  -0.000000E+00
0.940602E+00   0.000000E+00   0.100000E+01   0.000000E+00  -0.000000E+00

angle               s-sub-1                              s-sub-2
30.00   0.929626E-07   0.552326E-06   0.929626E-07   0.552326E-06
30.00   0.929522E-07   0.552301E-06   0.929522E-07   0.552301E-06

          intensity     deg of polzn
        0.313706E-12   0.000000E+00
        0.313677E-12   0.000000E+00

     s11              s12              s33              s34              pol
0.870508E+00   0.000000E+00   0.100000E+01   0.000000E+00  -0.000000E+00
0.870513E+00   0.000000E+00   0.100000E+01   0.000000E+00  -0.000000E+00

angle               s-sub-1                              s-sub-2
40.00   0.879815E-07   0.522731E-06   0.879815E-07   0.522731E-06
40.00   0.879718E-07   0.522709E-06   0.879718E-07   0.522709E-06

          intensity     deg of polzn
        0.280989E-12   0.000000E+00
        0.280964E-12   0.000000E+00

     s11              s12              s33              s34              pol
0.779721E+00   0.000000E+00   0.100000E+01   0.000000E+00  -0.000000E+00
0.779728E+00   0.000000E+00   0.100000E+01   0.000000E+00  -0.000000E+00

angle               s-sub-1                              s-sub-2
50.00   0.818409E-07   0.486247E-06   0.818409E-07   0.486247E-06
50.00   0.818321E-07   0.486228E-06   0.818321E-07   0.486228E-06

          intensity     deg of polzn
        0.243134E-12   0.000000E+00
        0.243114E-12   0.000000E+00

     s11              s12              s33              s34              pol
0.674679E+00   0.000000E+00   0.100000E+01   0.000000E+00  -0.000000E+00
0.674688E+00   0.000000E+00   0.100000E+01   0.000000E+00  -0.000000E+00

angle               s-sub-1                              s-sub-2
60.00   0.747273E-07   0.443982E-06   0.747273E-07   0.443982E-06
60.00   0.747194E-07   0.443966E-06   0.747194E-07   0.443966E-06

          intensity     deg of polzn
        0.202705E-12  -0.186801E-15
        0.202689E-12   0.000000E+00

     s11              s12              s33              s34              pol
0.562489E+00  -0.378653E-28   0.100000E+01  -0.311334E-16   0.186801E-15
```

```
 0.562500E+00  0.000000E+00  0.100000E+01  0.000000E+00 -0.000000E+00

 angle           s-sub-1                            s-sub-2
 70.00   0.668568E-07  0.397221E-06  0.668568E-07  0.397221E-06
 70.00   0.668499E-07  0.397208E-06  0.668499E-07  0.397208E-06

           intensity    deg of polzn
         0.162254E-12 -0.311160E-15
         0.162243E-12  0.000000E+00

      s11           s12           s33           s34           pol
 0.450243E+00 -0.504871E-28  0.100000E+01 -0.388950E-16  0.311160E-15
 0.450255E+00  0.000000E+00  0.100000E+01  0.000000E+00 -0.000000E+00

 angle           s-sub-1                            s-sub-2
 80.00   0.584687E-07  0.347384E-06  0.584687E-07  0.347384E-06
 80.00   0.584628E-07  0.347373E-06  0.584628E-07  0.347373E-06

           intensity    deg of polzn
         0.124094E-12 -0.305134E-15
         0.124086E-12  0.000000E+00

      s11           s12           s33           s34           pol
 0.344352E+00 -0.378653E-28  0.100000E+01 -0.762834E-16  0.305134E-15
 0.344363E+00  0.000000E+00  0.100000E+01  0.000000E+00 -0.000000E+00

 angle           s-sub-1                            s-sub-2
 90.00   0.498177E-07  0.295986E-06  0.498177E-07  0.295986E-06
 90.00   0.498129E-07  0.295977E-06  0.498129E-07  0.295977E-06

           intensity    deg of polzn
         0.900892E-13 -0.700515E-15
         0.900839E-13  0.000000E+00

      s11           s12           s33           s34           pol
 0.249991E+00 -0.631089E-28  0.100000E+01 -0.210155E-15  0.700515E-15
 0.250000E+00  0.000000E+00  0.100000E+01  0.000000E+00 -0.000000E+00

 angle           s-sub-1                            s-sub-2
100.00   0.411668E-07  0.244587E-06  0.411668E-07  0.244587E-06
100.00   0.411630E-07  0.244581E-06  0.411630E-07  0.244581E-06

           intensity    deg of polzn
         0.615177E-13 -0.923279E-15
         0.615145E-13  0.000000E+00

      s11           s12           s33           s34           pol
 0.170707E+00 -0.567980E-28  0.100000E+01 -0.230820E-15  0.923279E-15
 0.170714E+00  0.000000E+00  0.100000E+01  0.000000E+00 -0.000000E+00

 angle           s-sub-1                            s-sub-2
110.00   0.327789E-07  0.194751E-06  0.327789E-07  0.194751E-06
```

```
110.00  0.327759E-07  0.194747E-06  0.327759E-07  0.194747E-06

          intensity     deg of polzn
        0.390025E-13 -0.970843E-15
        0.390007E-13  0.000000E+00

     s11           s12           s33           s34           pol
  0.108229E+00 -0.378653E-28  0.100000E+01 -0.303389E-15  0.970843E-15
  0.108234E+00  0.000000E+00  0.100000E+01  0.000000E+00 -0.000000E+00

 angle           s-sub-1                       s-sub-2
120.00  0.249086E-07  0.147991E-06  0.249086E-07  0.147991E-06
120.00  0.249065E-07  0.147989E-06  0.249065E-07  0.147989E-06

          intensity     deg of polzn
        0.225219E-13 -0.133100E-14
        0.225210E-13  0.000000E+00

     s11           s12           s33           s34           pol
  0.624965E-01 -0.299767E-28  0.100000E+01 -0.385291E-15  0.133100E-14
  0.625000E-01  0.000000E+00  0.100000E+01  0.000000E+00 -0.000000E+00

 angle           s-sub-1                       s-sub-2
130.00  0.177953E-07  0.105728E-06  0.177953E-07  0.105728E-06
130.00  0.177938E-07  0.105727E-06  0.177938E-07  0.105727E-06

          intensity     deg of polzn
        0.114952E-13 -0.240189E-14
        0.114948E-13  0.000000E+00

     s11           s12           s33           s34           pol
  0.318982E-01 -0.276101E-28  0.100000E+01 -0.720567E-15  0.240189E-14
  0.319002E-01  0.000000E+00  0.100000E+01  0.000000E+00 -0.000000E+00

 angle           s-sub-1                       s-sub-2
140.00  0.116550E-07  0.692465E-07  0.116550E-07  0.692465E-07
140.00  0.116540E-07  0.692455E-07  0.116540E-07  0.692455E-07

          intensity     deg of polzn
        0.493091E-14 -0.399957E-14
        0.493076E-14  0.000000E+00

     s11           s12           s33           s34           pol
  0.136829E-01 -0.197215E-28  0.100000E+01 -0.119987E-14  0.399957E-14
  0.136838E-01  0.000000E+00  0.100000E+01  0.000000E+00 -0.000000E+00

 angle           s-sub-1                       s-sub-2
150.00  0.667421E-08  0.396539E-07  0.667421E-08  0.396539E-07
150.00  0.667367E-08  0.396534E-07  0.667367E-08  0.396534E-07

          intensity     deg of polzn
        0.161698E-14 -0.701301E-14
```

```
       0.161693E-14  0.000000E+00

     s11            s12            s33            s34            pol
 0.448698E-02 -0.113399E-28  0.100000E+01 -0.210390E-14  0.701301E-14
 0.448730E-02  0.000000E+00  0.100000E+01  0.000000E+00 -0.000000E+00

 angle          s-sub-1                      s-sub-2
160.00  0.300433E-08  0.178498E-07  0.300433E-08  0.178498E-07
160.00  0.300409E-08  0.178496E-07  0.300409E-08  0.178496E-07

         intensity    deg of polzn
       0.327641E-15 -0.166282E-13
       0.327633E-15  0.000000E+00

     s11            s12            s33            s34            pol
 0.909178E-03 -0.544807E-29  0.100000E+01 -0.498469E-14  0.166282E-13
 0.909245E-03  0.000000E+00  0.100000E+01  0.000000E+00 -0.000000E+00

 angle          s-sub-1                      s-sub-2
170.00  0.756830E-09  0.449660E-08  0.756830E-09  0.449660E-08
170.00  0.756770E-09  0.449656E-08  0.756770E-09  0.449656E-08

         intensity    deg of polzn
       0.207922E-16 -0.669882E-13
       0.207918E-16  0.000000E+00

     s11            s12            s33            s34            pol
 0.576968E-04 -0.139283E-29  0.100000E+01 -0.200446E-13  0.669882E-13
 0.577011E-04  0.000000E+00  0.100000E+01  0.000000E+00 -0.000000E+00


degree of polarization undefined for   180.0000000000000     deg.(sx)

 angle          s-sub-1                      s-sub-2
180.00 -0.198521E-22  0.158818E-21  0.198521E-22 -0.158818E-21
180.00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00

         intensity    deg of polzn
       0.256172E-43  0.000000E+00
       0.000000E+00  0.000000E+00

     s11            s12            s33            s34            pol
 0.710856E-31  0.000000E+00 -0.100000E+01  0.000000E+00 -0.000000E+00
 0.277519E+13  0.000000E+00  0.000000E+00  0.000000E+00 -0.000000E+00


                             extinction    scattering    absorption
     efficiency factors  0.398549E-02  0.480485E-08  0.398549E-02
     asymmetry factor = 0.500006       backscatter =  0.102469E-38

                             extinction    scattering    absorption
     efficiency factors  0.398503E-02  0.480448E-08  0.398503E-02
```

```
     asymmetry factor = 0.500006       backscatter =  0.000000E+00

mie size parameter =  10.00000     permittivity =   0.224E+01  -0.300E+00
                                   permeability =   0.224E+01  -0.300E+00


 angle            s-sub-1                          s-sub-2
  0.00  0.604902E+02 -0.800379E+01  0.604902E+02 -0.800379E+01

          intensity    deg of polzn
        0.372313E+04  0.000000E+00

     s11           s12           s33           s34           pol
 0.100000E+01  0.000000E+00  0.100000E+01  0.000000E+00 -0.000000E+00

 angle            s-sub-1                          s-sub-2
 10.00  0.364065E+02 -0.285146E+01  0.364065E+02 -0.285146E+01

          intensity    deg of polzn
        0.133357E+04  0.000000E+00

     s11           s12           s33           s34           pol
 0.358185E+00  0.000000E+00  0.100000E+01 -0.213125E-16 -0.000000E+00

 angle            s-sub-1                          s-sub-2
 20.00  0.255793E+00  0.305124E+01  0.255793E+00  0.305124E+01

          intensity    deg of polzn
        0.937548E+01 -0.189468E-15

     s11           s12           s33           s34           pol
 0.251817E-02 -0.177636E-14  0.100000E+01 -0.710507E-15  0.189468E-15

 angle            s-sub-1                          s-sub-2
 30.00 -0.650130E+01  0.601006E+00 -0.650130E+01  0.601006E+00

          intensity    deg of polzn
        0.426280E+02  0.000000E+00

     s11           s12           s33           s34           pol
 0.114495E-01  0.000000E+00  0.100000E+01 -0.135431E-15 -0.000000E+00

 angle            s-sub-1                          s-sub-2
 40.00  0.170607E+01 -0.232783E+01  0.170607E+01 -0.232783E+01

          intensity    deg of polzn
        0.832946E+01 -0.213262E-15

     s11           s12           s33           s34           pol
 0.223722E-02 -0.177636E-14  0.100000E+01  0.159946E-15  0.213262E-15

 angle            s-sub-1                          s-sub-2
```

```
50.00  0.223135E+01  0.134911E+00  0.223135E+01  0.134911E+00

         intensity    deg of polzn
       0.499713E+01  0.444344E-15

     s11           s12           s33           s34            pol
0.134219E-02  0.222045E-14  0.100000E+01 -0.466561E-15 -0.444344E-15

angle            s-sub-1                      s-sub-2
60.00 -0.976135E+00  0.172588E+01 -0.976135E+00  0.172588E+01

         intensity    deg of polzn
       0.393152E+01 -0.564781E-15

     s11           s12           s33           s34            pol
0.105597E-02 -0.222045E-14  0.100000E+01 -0.564781E-16  0.564781E-15

angle            s-sub-1                      s-sub-2
70.00 -0.105808E+01 -0.219556E+00 -0.105808E+01 -0.219556E+00

         intensity    deg of polzn
       0.116774E+01 -0.570447E-15

     s11           s12           s33           s34            pol
0.313645E-03 -0.666134E-15  0.100000E+01 -0.570447E-15  0.570447E-15

angle            s-sub-1                      s-sub-2
80.00  0.166837E+00 -0.116371E+01  0.166837E+00 -0.116371E+01

         intensity    deg of polzn
       0.138205E+01 -0.722984E-15

     s11           s12           s33           s34            pol
0.371207E-03 -0.999201E-15  0.100000E+01  0.140580E-15  0.722984E-15

angle            s-sub-1                      s-sub-2
90.00  0.647575E+00 -0.683249E-01  0.647575E+00 -0.683249E-01

         intensity    deg of polzn
       0.424022E+00 -0.202919E-14

     s11           s12           s33           s34            pol
0.113889E-03 -0.860423E-15  0.100000E+01 -0.474570E-15  0.202919E-14

angle            s-sub-1                      s-sub-2
100.00  0.342863E+00  0.567668E+00  0.342863E+00  0.567668E+00

         intensity    deg of polzn
       0.439801E+00 -0.258748E-14

     s11           s12           s33           s34            pol
0.118127E-03 -0.113798E-14  0.100000E+01  0.100975E-14  0.258748E-14
```

```
 angle              s-sub-1                             s-sub-2
110.00 -0.204251E+00  0.304099E+00 -0.204251E+00  0.304099E+00

          intensity    deg of polzn
        0.134195E+00 -0.320587E-14

    s11              s12              s33              s34              pol
 0.360436E-04 -0.430211E-15  0.100000E+01 -0.149952E-14  0.320587E-14

 angle              s-sub-1                             s-sub-2
120.00 -0.348717E+00  0.223846E-01 -0.348717E+00  0.223846E-01

          intensity    deg of polzn
        0.122104E+00 -0.681930E-14

    s11              s12              s33              s34              pol
 0.327962E-04 -0.832667E-15  0.100000E+01 -0.161248E-14  0.681930E-14

 angle              s-sub-1                             s-sub-2
130.00 -0.134797E+00 -0.107422E+00 -0.134797E+00 -0.107422E+00

          intensity    deg of polzn
        0.297099E-01 -0.648115E-14

    s11              s12              s33              s34              pol
 0.797983E-05 -0.192554E-15  0.100000E+01 -0.992608E-15  0.648115E-14

 angle              s-sub-1                             s-sub-2
140.00 -0.461748E-02 -0.147281E+00 -0.461748E-02 -0.147281E+00

          intensity    deg of polzn
        0.217131E-01 -0.511316E-14

    s11              s12              s33              s34              pol
 0.583194E-05 -0.111022E-15  0.100000E+01 -0.320072E-14  0.511316E-14

 angle              s-sub-1                             s-sub-2
150.00  0.560250E-02 -0.764339E-01  0.560250E-02 -0.764339E-01

          intensity    deg of polzn
        0.587352E-02 -0.826970E-14

    s11              s12              s33              s34              pol
 0.157758E-05 -0.485723E-16  0.100000E+01  0.117216E-14  0.826970E-14

 angle              s-sub-1                             s-sub-2
160.00  0.258230E-01 -0.216056E-02  0.258230E-01 -0.216056E-02

          intensity    deg of polzn
        0.671494E-03  0.161461E-14

    s11              s12              s33              s34              pol
```

```
0.180358E-06  0.108420E-17  0.100000E+01  0.219385E-13 -0.161461E-14

angle              s-sub-1                            s-sub-2
170.00  0.203439E-01  0.744313E-02  0.203439E-01  0.744313E-02

        intensity    deg of polzn
       0.469274E-03 -0.247211E-13

    s11           s12           s33           s34           pol
0.126043E-06 -0.116010E-16  0.100000E+01  0.488645E-13  0.247211E-13

angle              s-sub-1                            s-sub-2
180.00  0.726314E-15 -0.526075E-15 -0.726314E-15  0.526075E-15

        intensity    deg of polzn
       0.804287E-30  0.000000E+00

    s11           s12           s33           s34           pol
0.216025E-33  0.000000E+00 -0.100000E+01  0.000000E+00 -0.000000E+00


                          extinction    scattering    absorption
    efficiency factors  0.241961E+01  0.116202E+01  0.125759E+01
    asymmetry factor = 0.958194       backscatter =  0.321715E-31
```